# Calculating proper motions in the WFCAM Science Archive for the UKIRT Infrared Deep Sky Surveys

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**Abstract.** The ninth data release from the UKIRT Infrared Deep Sky Surveys (hereafter UKIDSS DR9), represents five years worth of observations by its wide–field camera (WFCAM) and will be the first to include proper motion values in its source catalogues for the shallow, wide–area surveys; the Large Area Survey (LAS), Galactic Clusters Survey (GCS) and (ultimately) Galactic Plane Survey (GPS). We, the Wide Field Astronomy Unit (WFAU) at the University of Edinburgh who prepare these regular data releases in the WFCAM Science Archive (WSA), describe in this paper how we make optimal use of the individual detection catalogues from each observation to derive high–quality astrometric fits for the positions of each detection enabling us to calculate a proper motion solution across multiple epochs and passbands when constructing a merged source catalogue. We also describe how the proper motion solutions affect the calculation of the various attributes provided in the database source catalogue tables, what measures of data quality we provide and a demonstration of the results for observations of the Pleiades cluster.

## 1. Proper motions in the context of the WSA

The proper motions are calculated during the formation of the passband-merged source catalogue (Hambly et al. 2008), and thus are limited by the matching tolerance used in this process of 2". This limits measurement of very high proper motions, but > 99% of (generally small) stellar motions can be measured, down to an astrometric precision of  $\sim 5 \text{ ms/yr}$  (milliarcseconds per year) for brighter stars with the full (currently) 5-year baseline. To improve their reliability, proper motion solutions are only calculated if any pair of individual observed detections are separated by at least half a year, resulting in the largest measurable proper motion of 4''/year in the most favourable circumstances.

## 2. Astrometry optimisations

To calculate useful proper motions we need the relative astrometry to be as accurate as possible. Corrections were calculated for each overlapping frame that makes up the set of observations in the passband–merged catalogue to account for systematic errors.

Our implementation works everywhere in local plane coordinates with an arbitrary tangent point chosen to be that in the centre of the detector frame set being processed. If  $(\xi, \eta)$  are untransformed slave coordinates, then for every slave frame we compute an array of linear coefficients  $(a \dots f)$  such that  $\xi' = a + b\xi + c\eta$  and  $\eta' = d + e\xi + f\eta$ , where primed symbols denote coordinates transformed to the master (shortest wavelength) reference frame, via Singular Value Decomposition — using a C implementation in

SLALIB (Wallace 1994) — of the design matrix on the left-hand side of the matrix equation

| $\begin{pmatrix} 1\\ 0 \end{pmatrix}$ | ${\xi_1 \over 0}$ | $\eta_1 \ 0$ | 0<br>1 | $0 \ \xi_1$ | $\begin{pmatrix} 0\\ \eta_1 \end{pmatrix}$ | $\left(\begin{array}{c} a\\b\end{array}\right)$ |   | $\left(\begin{array}{c}\xi_1^M\\\eta_1^M\\\vdots\\\xi_N^M\\\eta_N^M\end{array}\right)$ |
|---------------------------------------|-------------------|--------------|--------|-------------|--|---|---|--|
|                                       |                   | ÷            | ÷      | ÷           | :  | $\begin{pmatrix} c \\ d \end{pmatrix}$          | = | :  |
| 1                                     | $\xi_N \ 0$       | $\eta_N$     | 0      | 0           | 0  | e   |   | $\xi_N^M$  |
| (0)                                   | 0                 | 0            | 1      | $\xi_N$     | $\eta_N$ )                                 | $\left( f \right)$                              |   | $(\eta_N^{\dot{M}})$   |

where  $(\xi^M, \eta^M)$  are the master coordinates for the same source detection in the master frame, for N stars in common. The system is solved in an outer iterative loop with  $3\sigma$ rejection (using the RMS of all points) to home in on a stable solution known colloquially as a 'systematic error map'. If there is no stable solution or the systematic error found in any frame in the set is too large then that entire frame set is rejected and no proper motions calculated.

We chose to use SVD of the design matrix (as opposed, for example, to a more common decomposition of the normal equations matrix) for numerical stability and simplicity in the implementation. The matrix is unit weighted in order that the solution be dominated by the most numerous, faint stars which essentially means the zero–point of the relative proper motions subsequently derived will be defined by the most distant stars (and hence the proper motions can be considered to be close to absolute in some useful sense).

We note that we find significant zero-point offsets between frames observed more than a few years apart. This is not surprising: the pipeline astrometry for WFCAM is based on relatively bright 'standards' from 2MASS, and we expect to see the effects of bulk motions of these stars with respect to much fainter (and generally much more distant) stars. Finally, we note that our procedure makes no attempt at a treatment of atmospheric dispersion (differential and/or chromatic or otherwise; we simply assume such effects to be minimal in our infrared data over angular scales of one WFCAM detector frame, i.e.  $\sim 13'$ ).

## 3. Proper motion solution

A proper motion solution is obtained through the astrometric fit to all matched detections. The right ascension and declination coordinates are transformed to a tangentplane projection and undergo a linear transformation to remove systematic errors in their observation frame using the coefficients derived in the process described in section 2. A linear model is used to fit the resulting set of  $(\xi, \eta)_n$  coordinates for the detections as described by  $\xi'_n = \xi_n + \mu_{\xi}t'_n$  and  $\eta'_n = \eta_n + \mu_{\eta}t'_n$ , where  $\mu$  is the proper motion. Here,  $t'_n = t_n - \bar{t}$ , where  $t_n$  is the epoch of observation for each detection and  $\bar{t}$ is the inverse-variance-weighted mean epoch of all observed detections of the source (database column epoch), with variance  $\sigma_n^2$ . This is provided on a source-by-source basis in order to quote zero-point positions with minimum variance.

The measurement error,  $\sigma$ , is derived during catalogue extraction of the detector image as  $(\sigma_x, \sigma_y)$  pixels. We take the RMS of these values, adding in quadrature and convert via the pixel scale, p, of the detector frame. This is because the centroid error estimates in x and y from the catalogue extractor are highly correlated, and there is an arbitrary rotation between the detector coordinate system (x, y) and the local tangent plane  $(\xi, \eta)$  coordinate system (although for WFCAM mounted on UKIRT, this rotation is always very small).

We found during testing that the resulting distribution of  $\chi^2$  values for the astrometric fit of all proper motion solutions in the LAS clearly showed that the errors were underestimated for brighter stars. To counter this effect we introduced a minimum error that we empirically found to be optimal at a value of  $\sigma_{\min} = 10$  mas. Our justification for the *post-hoc* adjustment of the noise model is that we believe there to be a minimum centroiding error for the combination of the atmosphere, UKIRT and WFCAM that cannot be breached, regardless of arbitrarily increasing signal-to-noise ratio. Hence, we have

$$\sigma = \sqrt{\sigma_{\min}^2 + \frac{p^2}{2}(\sigma_x^2 + \sigma_y^2)}.$$

A weighted design matrix is formed to solve, again via SVD, the series of linear equations describing the proper motion in the position of the detections, where *N* is the number of observed detections of this source (database column nFrames):

$$\begin{pmatrix} 1/\sigma_1 & t'_1/\sigma_1 & 0 & 0\\ 0 & 0 & 1/\sigma_1 & t'_1/\sigma_1\\ \vdots & \vdots & \vdots & \vdots\\ 1/\sigma_N & t'_N/\sigma_N & 0 & 0\\ 0 & 0 & 1/\sigma_N & t'_N/\sigma_N \end{pmatrix} \begin{pmatrix} \bar{\xi}\\ \mu_{\xi}\\ \bar{\eta}\\ \mu_{\eta} \end{pmatrix} = \begin{pmatrix} \xi_1/\sigma_1\\ \eta_1/\sigma_1\\ \vdots\\ \xi_N/\sigma_N\\ \eta_N/\sigma_N \end{pmatrix}$$

This provides  $(\bar{\xi}, \bar{\eta})$ , the interpolated position of the source at epoch  $\bar{t}$  according to the linear model fit, and the proper motions  $(\mu_{\xi}, \mu_{\eta})$ . The source position in the database source catalogue table is determined by deprojecting the  $\bar{\xi}, \bar{\eta}$ ) coordinates back into celestial coordinates for right ascension and declination (columns ra, dec). The proper motions are also transformed to celestial coordinates (columns muRa, muDec) and the formal error estimates on all these astrometric parameter measurements (columns sigRa, sigDec, sigMuRa, sigMuDec) are propagated in the standard way via covariance. A goodness–of–fit of the resulting astrometric solution is then made with a reduced  $\chi^2$  test if there are at least 3 detections contributing to the solution (database column chi2).

If a proper motion solution is not possible for a particular source due to there not being at least two observed detections separated by half a year, or there were systematic errors in the astrometry of the observation frame that were too large, then the source positions are given by the detection observed at the shortest wavelength (the standard method as described in Hambly et al. (2008)) and the proper motion attributes are left as default values.

## 4. Proper motions in the Pleiades

The Galactic Clusters Survey (GCS) provided a rigorous test of the resulting proper motion values with observations of the well–studied Pleiades star cluster (Lodieu, Deacon & Hambly, in preparation). One of the most useful discriminants between field stars and cluster members for many of the GCS targets is that member stars exhibit peculiar motions relative to the field stars. This is illustrated in figure 1, which shows a proper motion vector-point diagram for all stars in the Pleiades field from the GCS DR9 data (the selection is based on good quality point source detections having J < 16.5). In this figure, a number of kinematic effects are apparent. Most significantly for the GCS, the members of the Pleiades form a distinct group at (+20, -40), the known cluster proper motion, opening up the possibility of proper motion selection and/or statistical analysis via proper motion membership probabilities. Furthermore, the general motion of the field stars is apparent as a distribution of points centred approximately on zero since the most numerous, faintest stars are generally at much larger distances such that their motions cannot be measured. This field star distribution is asymmetrical around (0,0)however, due to the general drift of the stars in the Galactic disk and the peculiar solar motion creating 'reflex' proper motions in more nearby stars. This can be seen in the figure as a greater number of points scattered into the south-east quadrant (as it happens, the peculiar Pleiades proper motion is in a direction in the vector point diagram that is near to the antapex of the solar motion). The typical errors in these proper motions, as measured from WFCAM frames taken over a period of  $\sim$  5 years, can be measured from the dispersion of the clump of Pleiades stars (since internal cluster motions will not contribute significantly to their proper motions at this level of accuracy). This dispersion is measured as ~ 4 mas/yr in either coordinate (for J < 16.5) and is a testament to the excellent imaging quality delivered by UKIRT/WFCAM and the accuracy of the processing software in the VDFS.



Figure 1. Plotting all proper motion values in the region of the Pleiades clearly identifies the cluster members through a common proper motion centred around  $\mu_{\alpha} = 20 \text{ mas/yr} \& \mu_{\delta} = -40 \text{ mas/yr}.$ 

#### References

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